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Draw a cross through the box (X) if you have NOT written in this booklet

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SCHOLARSHIP EXEMPLAR



Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Scholarship 2024 Calculus

Time allowed: Three hours
Total score: 32

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

Pull out Formulae and Tables Booklet S–CALCF from the centre of this booklet.

Show ALL working. Correct answers only will not be sufficient.

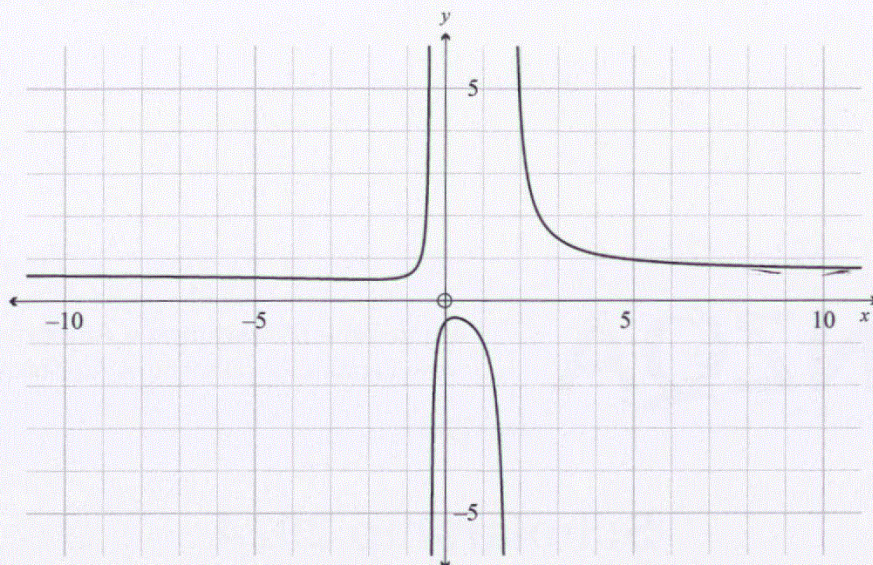
Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (///). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION ONE

- (a) Consider the curve $y = \frac{2x^2+1}{3x^2-4x-2}$, which is shown below.



- (i) Find the coordinates of any stationary points on the curve, and determine their nature.

$$y = \frac{2x^2+1}{3x^2-4x-2} \quad f = 2x^2+1 \quad f' = 4x$$

$$g = 3x^2-4x-2 \quad g' = 6x-4$$

$$\frac{dy}{dx} = \frac{(3x^2-4x-2)(4x) - (2x^2+1)(6x-4)}{(3x^2-4x-2)^2}$$

$$\begin{aligned} \frac{dy}{dx} = 0 &= (3x^2-4x-2)(4x) - (2x^2+1)(6x-4) \\ &= 12x^3 - 16x^2 - 8x - (12x^3 + 6x - 8x^2 - 4) \\ &= -16x^2 + 8x^2 - 8x - 6x + 4 = 0 \\ &= -8x^2 - 14x + 4 = 0 \end{aligned}$$

$$-4x^2 - 7x + 2 = 0$$

$$x = \frac{-7 \pm \sqrt{49+32}}{8} \quad \text{retry } \downarrow$$

$$y = \frac{2x^2+1}{3x^2-4x-2} \quad f = 2x^2+1 \quad f' = 4x$$

$$g = 3x^2-4x-2 \quad g' = 6x-4$$

$$\frac{dy}{dx} = \frac{4x(3x^2-4x-2) - (6x-4)(2x^2+1)}{(3x^2-4x-2)^2} = 0$$

$$0 = 12x^3 - 16x^2 - 8x - 12x^3 - 6x + 8x^2 + 4$$

$$= -8x^2 - 14x + 4$$

$$= -4x^2 - 7x + 2$$

$$= -(4x-1)(x+2)$$

$$x = \frac{1}{4} \quad \text{or} \quad x = -2$$

local minimum
↗

$$\text{wordmarks are } \left(\frac{1}{4}, -\frac{2}{5}\right) \text{ and } \left(-2, \frac{1}{2}\right)$$

local maximum.

- (ii) Find the value of $\lim_{x \rightarrow \infty} \frac{2x^2 + 1}{3x^2 - 4x - 2}$.

Hence, find the coordinate(s) where the curve intersects its own asymptote(s).

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 1}{3x^2 - 4x - 2} = \frac{2}{3} \quad \text{a asymptote} = y = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2x^2 + 1}{3x^2 - 4x - 2} \quad \cancel{6x^2} + 3 = \cancel{6x^2} - 8x - 4$$

$$8x + 7 = 0$$

$$8x = -7$$

$$x = -\frac{7}{8} \quad y = \frac{2}{3} \quad \left(-\frac{7}{8}, \frac{2}{3}\right)$$

two vertical asymptotes where $3x^2 - 4x - 2 = 0$

$$x = \frac{2 \pm \sqrt{10}}{3}, \quad \underline{\text{no intersections}}$$

- (b) For what real value(s) of a does the following system of equations have no real solutions?

$$x^3 + y^3 = 2$$

$$x + y = a$$

$$y = a - x \quad x^3 + (a - x)^3 = 2$$

$$\cancel{x^3} + a^3 - 3a^2x + 3ax^2 - \cancel{x^3} = 2$$

$$3ax^2 - 3a^2x + a^3 - 2 = 0$$

$$b^2 - 4ac < 0 \quad \text{no real solutions}$$

$$9a^4 - 4 \times 3a \times (a^3 - 2) < 0$$

$$9a^4 - 12a^4 + 12a < 0$$

$$-3a^4 + 12a < 0$$

$$a(-3a^3 + 12) < 0 \quad \underline{a < 0}$$

$$-3a^3 < -12$$

$$a^3 > 4$$

$$\underline{a > \sqrt[3]{4}}$$

so for values $a < 0$ and $a > \sqrt[3]{4}$

- (c) A landscaper is building steps up one side of a 48 metre high hill.

The slope of this side of the hill can be modelled by the curve

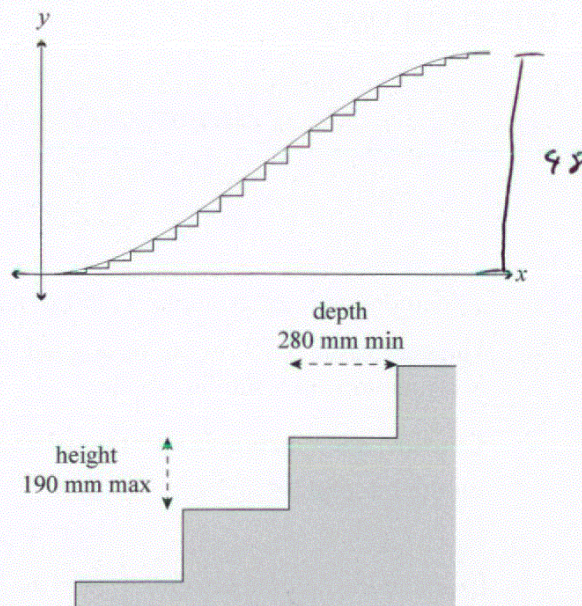
$$y = kx^2(126 - x)$$

where both x and y are in metres and k is a constant to be determined.

Building regulations state that each step needs to have a **minimum** depth of 280 mm and a **maximum** height of 190 mm, as shown.

If the landscaper chooses to build each step with a depth of 280 mm, would the steps all satisfy the maximum height regulations?

You must use calculus to support your answer.



$$\text{mm} \rightarrow \text{m} \quad 280 \text{ mm} = 0.28 \text{ m} \quad 190 \text{ mm} = 0.19 \text{ m}$$

$$\text{gradient of max stair} = \frac{0.19}{0.28} \approx \frac{19}{28}$$

$$y = kx^2(126 - x) = 126Kx^2 - Kx^3$$

$$\frac{dy}{dx} = 252Kx - 3Kx^2 = \text{gradient}$$

if max gradient is $< \frac{19}{28}$, the regulations will be satisfied

$$252Kx - 3Kx^2 = 0$$

$$x(252K - 3Kx) = 0$$

$$x = 0, \quad 3Kx = 252K$$

$$x = 84$$

$$x = 84$$

$$x = 84$$

$$\text{when } x = 84, y = 48$$

$$48 = k \cdot 84^2 (126 - 84)$$

$$k = \frac{48}{84^2 \cdot 42} = \frac{1}{6174}$$

$$48 = 84^2 k (126 - 84) \quad k = \frac{1}{6174}$$

$$\frac{d^2y}{dx^2} = 252K - 6Kx$$

$$\frac{d^2y}{dx^2} = 0$$

$$x = \frac{252}{6} = 42$$

$$\frac{dy}{dx} = \text{when } x = 42 = \frac{6}{7}$$

$$\text{as } \frac{6}{7} > \frac{19}{28}$$

all steps will be fine

QUESTION TWO

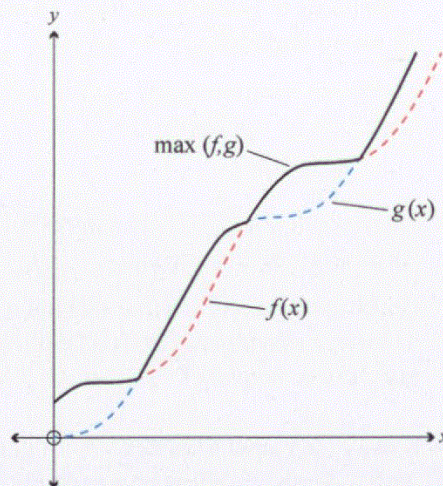
- (a) For any two functions f and g defined on an interval, $a \leq x \leq b$, the **max value** function is defined as:

$$\max(f, g) = \begin{cases} f(x) & \text{if } f(x) \geq g(x) \\ g(x) & \text{if } f(x) < g(x) \end{cases}$$

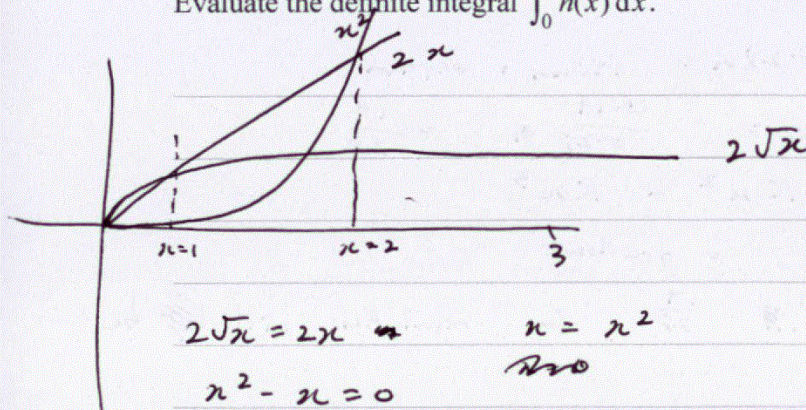
To demonstrate, an example is shown opposite.

The max value function can be applied to more than two functions in a similar manner.

Consider the function $h(x) = \max(2\sqrt{x}, 2x, x^2)$ on the interval $0 \leq x \leq 3$.



Evaluate the definite integral $\int_0^3 h(x) dx$.



$$2\sqrt{x} = 2x \quad x = x^2$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0 \quad x = 1$$

$$2x = x^2$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0 \quad x = 2$$

$$\text{so } 0 < x < 1 = 2\sqrt{x} \quad 1 < x < 2 = 2x \quad 2 < x \leq 3 = x^2$$

$$\int_0^3 h(x) dx = \int_0^1 2\sqrt{x} dx + \int_1^2 2x dx + \int_2^3 x^2 dx$$

$$= \left[\frac{4}{3} (x)^{\frac{3}{2}} \right]_0^1 + \left[x^2 \right]_1^2 + \left[\frac{x^3}{3} \right]_2^3$$

$$= \frac{4}{3} + 4 - 1 + 3^2 - \frac{8}{3}$$

$$= \frac{4}{3} + 12$$

$$= \frac{32}{3}$$

- (b) Given that $\frac{dy}{dx} = 1 + y^2$, find the value of M for which $\frac{d^3y}{dx^3} = M(1 + y^2)(1 + 3y^2)$.

$$f'(x) = 1 + f(x)^2$$

$$\frac{d^3y}{dx^3} = M(1 + 3y^2 + y^2 + 3y^4) = M(1 + 4y^2 + 3y^4)$$

$$\frac{d^3y}{dx^3} = M\left(\frac{dy}{dx}\right)\left(3\frac{dy}{dx} - 2\right)$$

$$\frac{d^2y}{dx^2} = 2y \, dy$$

$$dy = (1 + y^2) dx$$

$$\frac{d^2y}{dx^2} = 2(1 + y^2) = 2y + 2y^3$$

$$\begin{aligned} \frac{d^3y}{dx^3} &= \frac{d}{dx}(2y + 2y^3) \\ &= 2y(2 + 2y^2)(1 + y^2) \end{aligned}$$

$$\frac{d^3y}{dx^3} = (2 + 6y^2)(1 + y^2) = 2(1 + y^2)(1 + 3y^2)$$

$$\text{so } M = 2$$

- (c) For any real number x , the **absolute value** of x is defined as:

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Consider the function $s_n(x) = |x-1| + |x-2| + \dots + |x-n|$ with $n > 2$.

- (i) Find $s_3'\left(\frac{5}{2}\right)$, the derivative of $s_3(x)$ evaluated at $x = \frac{5}{2}$.

$$s_3(x) = |x-1| + |x-2| + |x-3| \quad \text{if } x = \frac{5}{2} \text{ we put in}$$

$$s_3(x) = |1.5| + |0.5| + |-0.5|$$

$$s_3(x) = x-1 + x-2 -x+3$$

$$s_3(x) = x \quad s_3' = 1$$

$$s_3'\left(\frac{5}{2}\right) = 1$$

- (ii) Find the minimum value of $s_{2024}(x)$.

Justify your answer.

$$s_{2024}(x) = |x-1| + |x-2| + \dots + |x-2024|$$

$$2024-1 = 2023 \quad \frac{2023}{2} = 1011.5$$

$$x = 1011.5$$

$$2024+1 = 2025 \quad \frac{2025}{2} = 1012.5$$

$$x = 1012.5$$

$$s_{2024}(1012.5) = (1011.5) \times 2024 = 2047226$$

for minimum value the average value should be lowest
in this case that average value x is in between
2024 and 1.

$$\frac{2u}{u+1}$$

$$2u(u+1)^{-1}$$

QUESTION THREE

- (a) Find the exact value of $\int_0^{\pi/4} \frac{\cos 2x}{\cos^2 x} dx$.

$$\cos^2 x = \cos 2x + \sin^2 x \quad | - \sin^2 x$$

$$2 \cos^2 x = \cos 2x + 1$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\int_0^{\pi/4} \frac{\cos 2x}{\cos^2 x} dx = \int_0^{\pi/4} \frac{2 \cos 2x}{\cos 2x + 1} dx$$

$$\int_0^{\pi/4} \frac{\cos 2x}{\cos^2 x} dx = \int_0^{\pi/4} \frac{\cos^2 x - \sin^2 x}{\cos^2 x} dx$$

$$= \int_0^{\pi/4} (1 - \tan^2 x) dx$$

$$= \int_0^{\pi/4} \frac{2 \cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} dx$$

$$= \int_0^{\pi/4} 2 - \sec^2 x dx$$

$$= [2x - \tan x]_0^{\pi/4} = \left[\frac{\pi}{2} - 1 \right] - 0$$

$$= \frac{\pi}{2} - 1$$

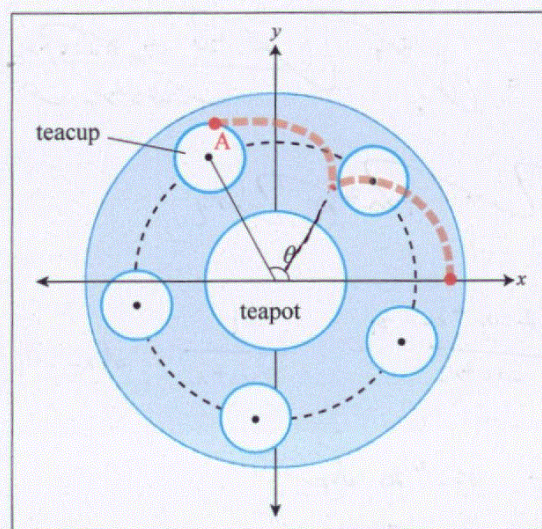
- (b) Alice takes a ride at an amusement park that involves
- five spinning teacups that rotate on circular saucers
 - a circular platform that rotates around a large teapot at its centre.

The coordinates of Alice's motion in the xy -plane (shown as point A in the diagram below) can be described by the parametric equations:

$$x = 4\cos\theta + \cos 4\theta$$

$$y = 4\sin\theta + \sin 4\theta$$

where θ is the anti-clockwise angle shown.



<https://www.flickr.com/photos/nycdc/4691066745>

- (i) Find the coordinates in the first quadrant where Alice is closest to the teapot.

Justify your answer.

$$\begin{aligned}
 D &= \text{distance to teapot} = \sqrt{x^2 + y^2} = \sqrt{(4\cos\theta + \cos 4\theta)^2 + (4\sin\theta + \sin 4\theta)^2} \\
 D^2 &= 16\cos^2\theta + 8\cos\theta\cos 4\theta + \cos^2 4\theta + 16\sin^2\theta + 8\sin\theta\sin 4\theta + \sin^2 4\theta \\
 &= 16(1) + 1 + 8(\cos\theta\cos 4\theta + \sin\theta\sin 4\theta) \\
 &= 17 + 8\left(\frac{\cos 5\theta + \cos 3\theta}{2} + \frac{\cos 3\theta - \cos 5\theta}{2}\right) \\
 &= 17 + 8\cos 3\theta
 \end{aligned}$$

$$\begin{aligned}
 \frac{dD}{d\theta} &= \frac{1}{2}(17 + 8\cos 3\theta)^{-\frac{1}{2}}(-24\sin 3\theta) \\
 &= -\frac{12\sin 3\theta}{(17 + 8\cos 3\theta)^{\frac{1}{2}}} \quad \frac{dD}{d\theta} = 0
 \end{aligned}$$

first quadrant is where $0 \leq \theta \leq \frac{\pi}{2}$

$$\theta = 1.047197551$$

$$x = 1.5$$

$$y = 2.598076211$$

coordinates are (1.5, 2.598076211)

- (ii) Find the distance Alice travels in one complete rotation around the teapot.

Note: when a curve is defined parametrically by the equations $x = f(\theta)$ and $y = g(\theta)$ on an interval $\alpha \leq \theta \leq \beta$, we can find its length, L , by using the formula:

$$L = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta$$

$$\frac{dx}{d\theta} = -4\sin\theta - 4\sin 4\theta = -4(\sin\theta + \sin 4\theta)$$

$$\frac{dy}{d\theta} = 4\cos\theta + 4\cos 4\theta = 4(\cos\theta + \cos 4\theta)$$

$$L = \int_0^{2\pi} \sqrt{4^2((\sin\theta + \sin 4\theta)^2 + (\cos\theta + \cos 4\theta)^2)} d\theta$$

$$4\sqrt{\sin^2\theta + 2\sin\theta\sin 4\theta + \sin^2 4\theta + \cos^2\theta + 2\cos\theta\cos 4\theta + \cos^2 4\theta}$$

$$= 4\sqrt{1 + 1 + 2(\sin\theta\sin 4\theta + \cos\theta\cos 4\theta)}$$

$$L = 4\int_0^{2\pi} \sqrt{2 + 2(\sin\theta\sin 4\theta + \cos\theta\cos 4\theta)} d\theta$$

$$= \sqrt{2 + \cos 3\theta - \cos 5\theta + \cos 5\theta + \cos 3\theta}$$

$$\sqrt{2 + 2\cos 3\theta}$$

$$\sqrt{2} \sqrt{1 + \cos 3\theta} = 2\sqrt{\cos^2 \frac{3\theta}{2}}$$

$$= 2\cos \frac{3\theta}{2}$$

$$L = 8 \int_0^{2\pi} \cos \frac{3\theta}{2} d\theta = 2 \times 8 \left[\frac{2}{3} \sin \frac{3\theta}{2} \right]_0^{2\pi}$$

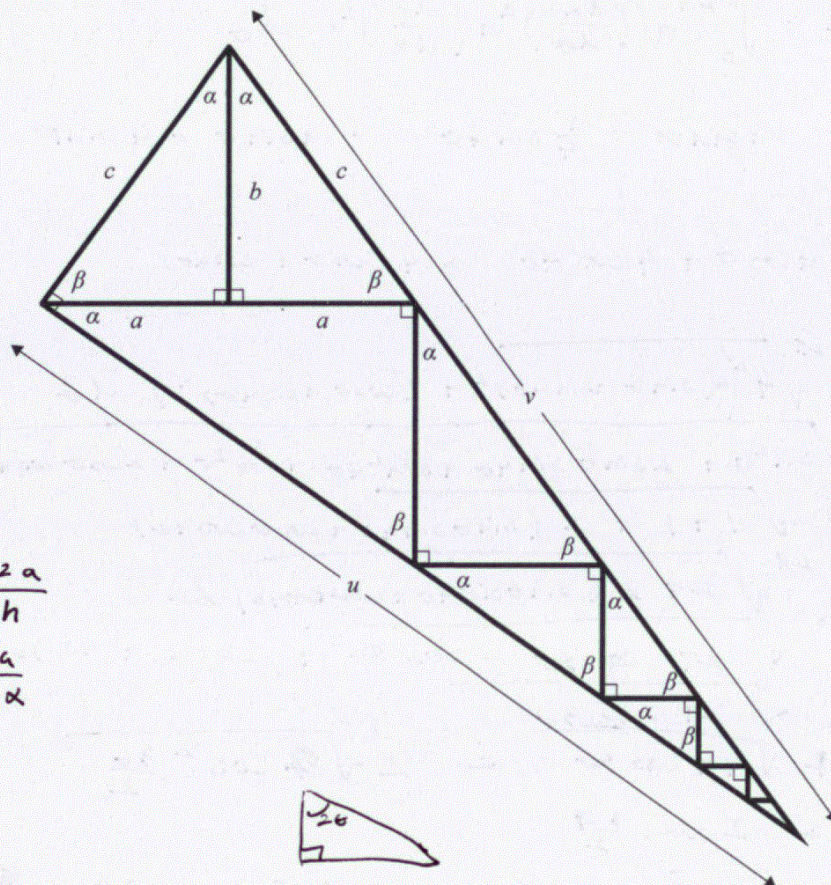
$$= 8 \times \frac{2}{3} \times \sin 3\pi = 0 \quad ?$$

$$= 2 \times 8 \times \frac{2}{3} = \frac{32}{3} = \frac{32}{3} \text{ units}$$

- (c) In 2023, two American high-school students discovered a new proof of the Pythagorean Theorem using trigonometry. In this question we will work through the key steps to derive their result.

The students' proof makes use of the diagram below, which consists of an infinite number of **similar** right-angled triangles enclosed within a large right-angled triangle. It was referred to by the students as a "waffle cone".

Note: to avoid circular logic, you should **not** make use of the Pythagorean Theorem or any of the Pythagorean trigonometric identities at any step in your working for this question.



$$\cos \alpha = \frac{2a}{h}$$

$$h = \frac{2a}{\cos \alpha}$$

- (i) Show that $c^2 = \frac{2ab}{\sin 2\alpha}$.

$$\sin 2\alpha = \frac{h}{c}$$

$$\frac{\sin \beta}{2a} = \frac{h}{c}$$

$$\frac{c}{b} = \frac{h}{2a}$$

$$\frac{c}{b} = \frac{2a}{\cos \alpha} \times \frac{1}{2a} = \frac{c}{b} = \frac{1}{\cos \alpha} \quad \text{||}$$

- (ii) Show that $u = \frac{2abc}{b^2 - a^2}$ and find a similar expression for v .

Hence, prove that $a^2 + b^2 = c^2$.

Hint: the following formulae will prove useful:

$$T_n = T_1 r^{n-1} \quad S_n = T_1 \left(\frac{1 - r^n}{1 - r} \right)$$

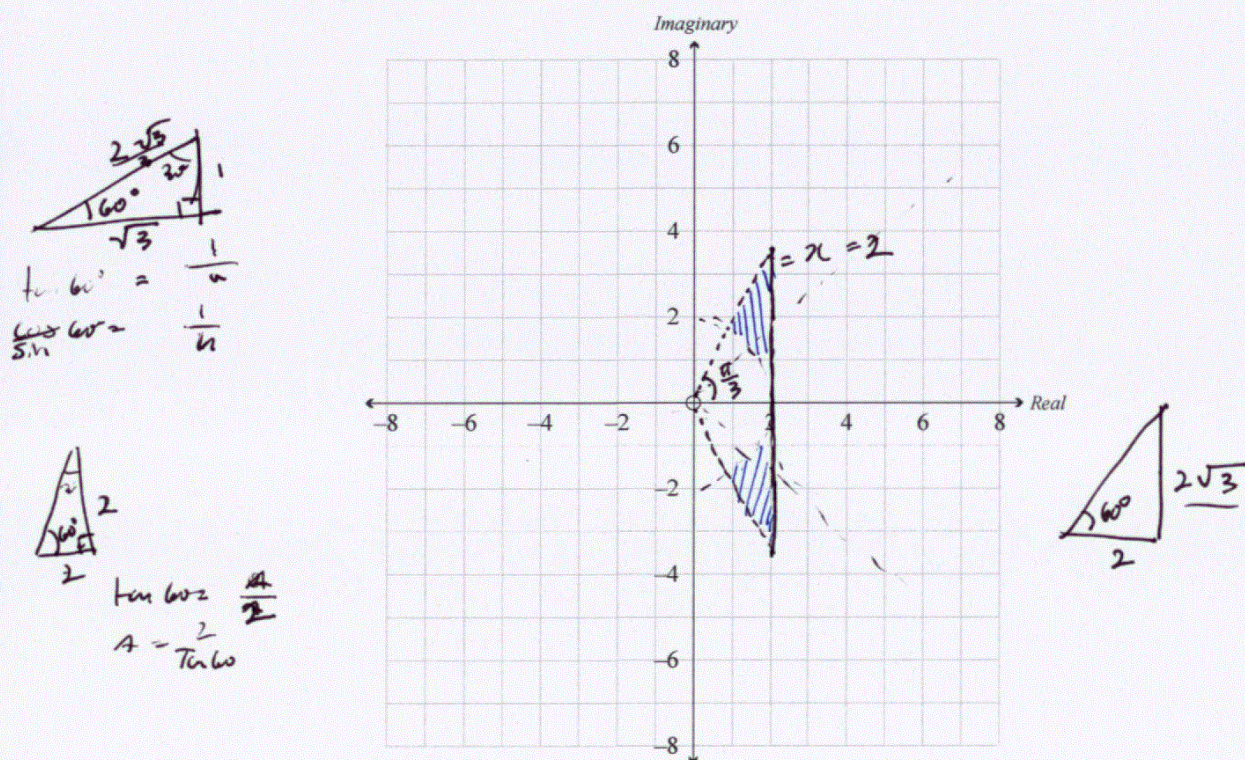
QUESTION FOUR

(a) Consider all the complex numbers z that satisfy all of the following three conditions:

- $-\frac{\pi}{3} \leq \arg(z) \leq \frac{\pi}{3}$
- $z + \bar{z} \leq 4$
- $|z| \geq 2$

Find the exact area of the region generated in an Argand diagram by the locus of points that represent z .

Use the Argand diagram below to support your working.



$\arg(z)$ is between $-\frac{\pi}{3}$ to $\frac{\pi}{3}$ $z = x + yi$

$$x + yi + x - yi = 4$$

$$2x = 4$$

$x = 2$ distance from centre must be more than 2

$$\text{Area of circle} = \pi r^2 = \pi 4$$

$$\pi 4 \times \frac{120}{360} = \frac{4}{3} \pi$$

$$\text{Area of triangles} = 2 \times \frac{1}{2} \times 4\sqrt{3} = 4\sqrt{3}$$

$$\text{Area of locus points} = 4\sqrt{3} - \frac{4}{3} \pi$$

$$= 4 \left(\sqrt{3} - \frac{1}{3} \pi \right) = 2.7394 (3025 \text{ units}^2)$$

(b) If $z = \cos\theta + i\sin\theta$ and $0 \leq \theta \leq \frac{\pi}{2}$, prove the following:

$$\begin{aligned}
 \text{(i)} \quad \frac{1}{1-z\cos\theta} &= 1 + i\cot\theta \\
 &= \frac{1}{1-2\cos\theta} = \frac{1}{1-\cos^2\theta + i\sin\theta\cos\theta} \times \frac{1-\cos^2\theta + i\sin\theta\cos\theta}{1-\cos^2\theta + i\sin\theta\cos\theta} \\
 &= \frac{1-\cos^2\theta + i\sin\theta\cos\theta}{(1-\cos^2\theta)^2 + \sin^2\theta\cos^2\theta} \\
 &= \frac{1-\cos^2\theta}{(1-\cos^2\theta)^2 + \sin^2\theta\cos^2\theta} + i \frac{\sin\theta\cos\theta}{(1-\cos^2\theta)^2 + \sin^2\theta\cos^2\theta} \\
 1-\cos^2\theta &= \sin^2\theta \\
 &= \frac{\sin^2\theta}{\sin^4\theta + \sin^2\theta\cos^2\theta} + i \frac{\sin\theta\cos\theta}{\sin^4\theta + \sin^2\theta\cos^2\theta} \\
 &= \frac{1}{\sin^2\theta + \cos^2\theta} + i \frac{\cos\theta}{\sin^2\theta} \frac{\sin\theta\cos\theta}{\sin^2\theta(\sin^2\theta + \cos^2\theta)} \\
 &= 1 + i \frac{\sin\theta\cos\theta}{\sin^2\theta} = 1 + i\cot\theta \quad \square
 \end{aligned}$$

(ii) $2\arg(z+1) = \arg(z)$

$$\begin{aligned}
 \arg(z) &= \theta \quad \Rightarrow \arg(z+1) = \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right) \\
 \frac{1}{2}\arg(z) &= \frac{\theta}{2} \\
 \arg(z+1) &= \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right) \quad \text{What I need,}
 \end{aligned}$$

$$\frac{\sin\theta}{1+\cos\theta} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1+2\cos^2\frac{\theta}{2}-1} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \quad \text{!!}$$

$$\begin{aligned}
 \arg(z+1) &= \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right) = \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) \\
 &= \frac{\theta}{2}
 \end{aligned}$$

$$2\arg(z+1) = \arg(z)$$

$$2 \times \frac{\theta}{2} = \theta$$

$$\theta = \theta \quad \square$$

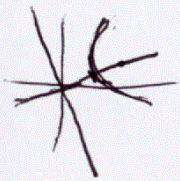
(c) The point $P(2p, p)$ is some point on the line $y = \frac{1}{2}x$ where $p \geq 0$.

- (i) Consider the locus of points that are the **same distance** from P as they are from the line $y = -2x$.

Explain why this locus is a parabola AND clearly describe its key features.

Note: you do **not** need to find the equation of the parabola.

~~As point P is on~~ because that is the definition of a parabola where its focus would be at point P , and its directrix ~~being~~ being the line $y = -2x$, the parabola would be off at an angle (normal to the directrix). The peak of the parabola would be the ~~halfway~~ halfway point to $0,0$ and the point P . ~~so~~ And as -2 is the normal of $\frac{1}{2}$ it would be $(P, \frac{P}{2})$

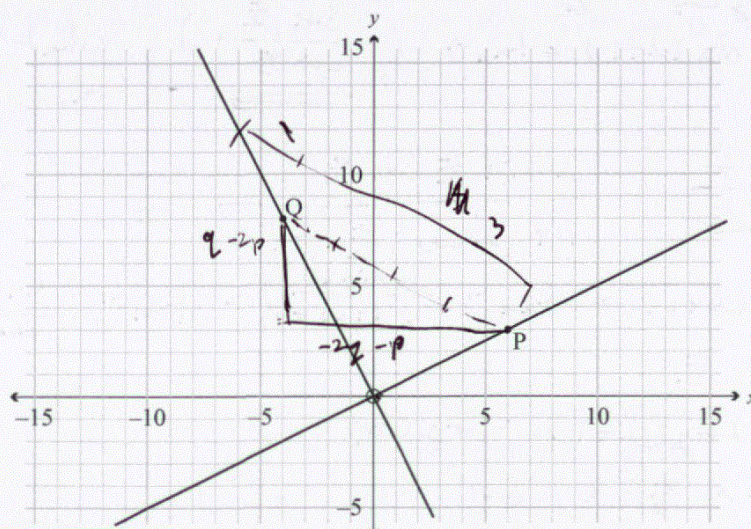


- (ii) Consider another point $Q(q, -2q)$ on the line $y = -2x$ where $q \leq 0$.

Let R be the region enclosed by the locus of points that are **three times as far** from P as they are from the **point** Q .

Now suppose the points P and Q are moving along their respective lines.

If p is increasing at a rate of 3 cm s^{-1} , and q is decreasing at a rate of 2 cm s^{-1} , at what rate is the area of the region R increasing when P is $(6, 3)$ and Q is $(-4, 8)$?



$$x_1, y_1 = 0, 0$$

$$x_2, y_2 = 2, 3$$

$$(x, y) (x_1, y_1)$$

$$(x_2, y_2)$$

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} = 3\sqrt{(x - x_2)^2 + (y - y_2)^2}$$

$$(x^2 + y^2 = 9(x^2 - 4x + 4 + y^2 - 8y + 16))$$

$$(x - x_1)^2 + (y - y_1)^2 = 9((x - x_2)^2 + (y - y_2)^2)$$

$$x^2 - 2xx_1 + x_1^2 + y^2 - 2yy_1 + y_1^2 = 9(x^2 - 2xx_2 + x_2^2 + y^2 - 2yy_2 + y_2^2)$$

$$9x^2 - 182xx_2 + 9x_2^2 + 9y^2 - 18yy_2 + 9y_2^2 - x^2 + 2xx_1 - x_1^2 - y^2 + 2yy_1 - y_1^2$$

$$= 8x^2 - 18xx_2 + 2xx_1 + 9x_2^2 - x_1^2 + 8y^2 - 18yy_2 + 2yy_1 + 9y_2^2 - y_1^2$$

anyways seems to be a ellipse or circle

distance from ~~the origin~~ are at the points noted in

$$\text{Distance to } P = \sqrt{(q - 2p)^2 + (-2q - p)^2} = D$$

$$\text{Diameter of circle} = \frac{D}{3} + \frac{D}{4} = \frac{7D}{12}$$

$$\text{radius} = \frac{7D}{6}$$

$$A = \text{Area} = \frac{7^2 \pi}{6^2} D^2$$

$$\frac{dp}{dt} = 3 \quad \frac{dq}{dt} = -2 \quad \left| \frac{dA}{dt} \right|$$

$$dp = 3dt \quad dq = -2dt$$

$$A = \frac{7^2 \pi}{6^2} ((q - 2p)^2 + (-2q - p)^2)$$

$$A = \frac{7^2 \pi}{6^2} (q^2 - 4qp + 4p^2 + 4q^2 + 4qp + p^2)$$

$$A = \frac{7^2 \pi}{6^2} (5q^2 + 5p^2) \quad dD = \frac{7^2 \pi}{6^2} \times 5 (2q dq + 2p dp)$$

$$= \frac{7^2 \pi \times 5 \times 2}{6^2} \times dt (q(-2) + 3p)$$

$$\frac{dA}{dt} = \frac{7^2 \pi \times 5 \times 2}{6^2} (-2q + 3p) = q = -4, p = 3$$

$$= \frac{4165}{18} \pi \cdot \text{cm}^2 \text{ s}^{-1}$$

Extra space if required.
Write the question number(s) if applicable.

QUESTION
NUMBER

1 a i)

$$\frac{dy}{dx} = \frac{-4x^2 - 7x + 2}{(3x^2 - 4x - 2)^2}$$

$$x = \frac{1}{4} \quad x = -2$$

~~$$\frac{d^2y}{dx^2} = f(x) = -4x^2 - 7x + 2$$~~

$$\frac{d^2y}{dx^2} = -2.2755 \quad x = \frac{1}{4}$$

$$\frac{d^2y}{dx^2} < 0 \quad \text{so is a local maximum}$$

$$\frac{d^2y}{dx^2} = 0.0555 > 0 \quad \text{is a local maximum} \quad x = -2$$

Scholarship

Subject: Calculus

Standard: 93202

Total score: 26

Q	Score	Marker commentary
1	6	The candidate showed ability in differentiating a rational function using quotient rule and determine the nature of the turning points in Q1(a)(i). They also showed proficiency in dealing with limit of a rational function and thereafter finding the intersection of the function with its horizontal asymptote in Q1(a)(ii). They found the greatest gradient of the function by finding the second derivative in Q1(c), although they should have checked the height of stairs at either side of this value.
2	7	The candidate demonstrated accuracy in identifying the maximum function as a step function of three functions in Q2(a). In Q2(b), they exhibited an excellent understanding of the implicit second derivative function. In Q2(c)(i), they successfully eliminated the absolute value sign and found the derivative of the function at a point. However, they struggled to extend this approach to handle the sum of 2024 absolute values in Q2(c)(ii).
3	6	The candidate displayed fluency in applying trigonometric identities throughout this question. They successfully found the exact value of the definite integral in Q3(a) and correctly simplified the complex trigonometric function before differentiating in Q3(b)(i) and integrating in Q3bii. However, their solution in Q39b)(ii) was incomplete as they did not substitute the correct limits, which would have allowed them to fully solve the problem.
4	7	The candidate demonstrated a thorough understanding of the concept of loci in various contexts, including complex numbers in Q4(a) and conics in Q4(c)(i). They also displayed aptitude in applying trigonometric identities to proof questions in the topic of complex numbers in Q4(b).