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TOP SCHOLAR

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New Zealand Qualifications Authority

Scholarship 2023
Calculus

Time allowed: Three hours
Total score: 32

ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write your answers in this booklet.

Make sure that you have Formulae Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

If in any question you make additions to a diagram and refer to those additions in your solution, the diagram must be replicated in this booklet as part of your solution.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area ( ). This area may be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

1(a)(i) P is on the circle $\Rightarrow (-8)^2 + (6)^2 = r^2 \Rightarrow r = 10.$ (assume $r>0$)

Additionally P is on hyperbola $\Rightarrow k = (-8)(6) = -48$

$x^2+y^2=100$

$x^2 + \frac{k^2}{x^2} = 100$

$x^2 + \frac{2304}{x^2} = 100$

$x^4 - 100x^2 + 2304 = 0$

$(x^2-64)(x^2-36) = 0$

$x = \pm 6, \pm 8.$

$x<0$, so $x = -6, -8.$

$Q: x = -6$ (since P is $x=-8$)

$y = \frac{k}{x} = \frac{-48}{-6} = 8.$ so $Q = (-6, 8)$

Gradient $QR = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8-0}{-6-10} = -\frac{1}{2}$ (since $R = (10,0)$)

$y - y_1 = m(x - x_1)$

$y = -\frac{1}{2}(x-10)$

$y = -\frac{1}{2}x + 5$

1(a)(ii) By inspection, gradient $RT = 1$.

We need to prove gradient $PQ = 1$.

Let $P = (a, b)$, where $a<0$ and $b>0$.

Consider the point $Q_2 = (-b, -a) = (x_{Q_2}, y_{Q_2})$.

Since P is on the circle, $a^2+b^2=r^2$.

Therefore, since $x_{Q_2}^2 + y_{Q_2}^2 = (-b)^2+(-a)^2 = a^2+b^2 = r^2$, Q_2 must also lie on the circle. Also, as P is on the hyperbola, $ab=k$, therefore Q_2 must lie on the hyperbola since $x_{Q_2}y_{Q_2} = (-b)(-a) = ab = k$.

Since $a<0$ and $b>0$, $x_{Q_2} = -b < 0$ and $y_{Q_2} = -a > 0$.

This information is enough to conclude that point Q_2 is ~~[illegible]~~ in fact point Q, as there are only two intersections in the second quadrant $(x_{Q_2} < 0;\ y_{Q_2} > 0)$ between the two curves.

Hence $Q = (-b, -a)$, and we calculate:

$$\text{gradient } PQ = \frac{y_P - y_Q}{x_P - x_Q} = \frac{b-(-a)}{a-(-b)} = \frac{a+b}{a+b} = 1,$$

as required. #

(Note: in the limiting case where $P = Q$, PQ is a tangent whose gradient is 1. Proof: limits)

1(b) $4\log_2(8x^3) + \log_5(y^6) = 17$ ... (1)

$\log_2(x^5) + \log_5(y^2) = 3$ ... (2)

Assume $x, y > 0$ for now.

justifies these steps

(1): $4\left[\log_2(8) + \log_2(x^3)\right] + 6\log_5(y) = 17$

$4\left[3 + 3\log_2(x)\right] + 6\log_5(y) = 17$

$12 + 12\log_2(x) + 6\log_5(y) = 17$

$12\log_2(x) + 6\log_5(y) = 5$ ... (3)

(2): $5\log_2(x) + 2\log_5(y) = 3$

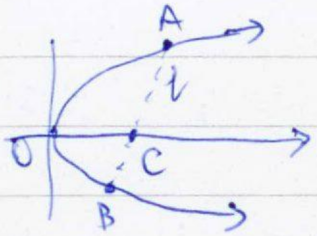
$\Rightarrow 15\log_2(x) + 6\log_5(y) = 9$ ... (4)

(4) − (3): $3\log_2(x) = 4 \Rightarrow \log_2(x) = \frac{4}{3}$ $(\Rightarrow x = 2^{\frac{4}{3}})$

Substitute back: $\log_5(y) = \frac{3 - 5\log_2(x)}{2} = \frac{-11}{6}$. $(\Rightarrow y = 5^{-\frac{11}{6}})$

We see that, assuming logarithms require positive inputs, $x^5 > 0 \Rightarrow x > 0$, however y can be negative and y^6 and y^2 would still be possible. It is at this point that we move beyond our initial assumption and conclude that all possible solution sets are $(x,y) = (2^{\frac{4}{3}}, 5^{-\frac{11}{6}})$ and $(x,y) = (2^{\frac{4}{3}}, -5^{-\frac{11}{6}})$.

1(c)



Start by defining point A parametrically as
$A = \left(\frac{ka^2}{4}, \frac{ka}{2}\right)$. $(a>0)$.

The gradient OA is $\frac{\frac{ka}{2}}{\frac{ka^2}{4}} = \frac{2}{a}$.

Hence gradient OB is $\frac{-a}{2}$ as $OB \perp OA$.

Let $B = \left(\frac{kb^2}{4}, \frac{kb}{2}\right)$. $(b<0)$

Gradient $OB = \frac{\frac{kb}{2}}{\frac{kb^2}{4}} = \frac{2}{b}$.

Hence $\frac{2}{b} = -\frac{a}{2} \Rightarrow ab = -4$.

Let line l cross the x-axis at $C: (c, 0)$.

By the collinearity, gradient AC = gradient AB.

$$\text{gradient } AB = \frac{\frac{ka}{2} - \frac{kb}{2}}{\frac{ka^2}{4} - \frac{kb^2}{4}} = 2\left(\frac{a-b}{a^2-b^2}\right) = \frac{2\cancel{(a-b)}}{\cancel{(a-b)}(a+b)} = \frac{2}{a+b}$$

$$\text{gradient } AC = \frac{\frac{ka}{2}}{\frac{ka^2}{4} - c}.$$

$$\frac{\frac{ka}{2}}{\frac{ka^2}{4} - c} = \frac{2}{a+b}$$

$$\Rightarrow \frac{2ka}{ka^2 - 4c} = \frac{2}{a+b}$$

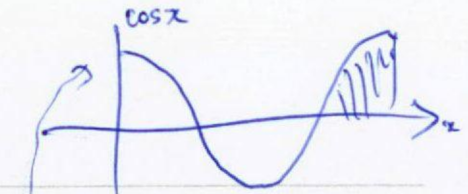
$$\Rightarrow ka(a+b) = ka^2 - 4c$$

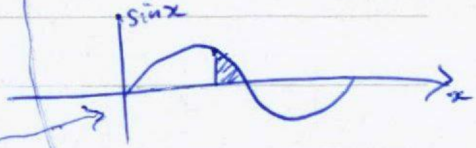
$$\Rightarrow \cancel{ka^2} + kab = \cancel{ka^2} - 4c$$

$$\Rightarrow kab = -4c$$

$$\Rightarrow -4k = -4c \Rightarrow c = k, \text{ as required.}$$

Hence l cross x-axis at $(k, 0)$. #





2(a) $\cot(x) = \frac{-5}{12} \Rightarrow \cot^2(x) = \frac{25}{144}$

$\Rightarrow 1 + \cot^2(x) = \frac{169}{144}$

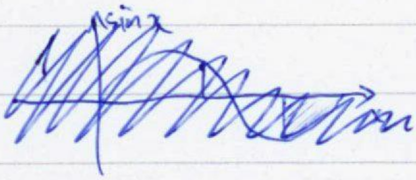
$\Rightarrow \operatorname{cosec}^2(x) = \frac{169}{144}$

$\Rightarrow \sin^2(x) = \frac{144}{169} \Rightarrow \cos^2(x) = 1 - \frac{144}{169}$

$= \frac{25}{169}$

~~$\Rightarrow \sin(x) = \pm\frac{12}{13}$~~ $\Rightarrow \cos x = \pm\frac{5}{13}$

but $\boxed{\frac{3\pi}{2} < x < 2\pi \Rightarrow \cos x > 0 \text{ so } \boxed{\cos x = \frac{5}{13}}}$



~~But as $\frac{3\pi}{2} < x < 2\pi$, graphically, $\sin x < 0$.~~

~~Hence $\sin x = -\frac{12}{13}$.~~

~~Hence $\sin(x) = -\frac{12}{13}$~~ $\rightarrow \cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\boxed{\frac{3\pi}{4} < \frac{x}{2} < \pi \Rightarrow \sin\frac{x}{2} > 0}$ $\Rightarrow \sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$

$= \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}$

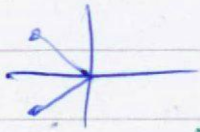
2(b) Notice that $z = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$,

as $\cos\left(\frac{2\pi}{3}\right) = -\cos\left(\pi - \frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$

and $\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{2\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

$z = \operatorname{cis}\left(\frac{2\pi}{3}\right)$

$\bar{z} = \operatorname{cis}\left(-\frac{2\pi}{3}\right)$



$(\bar{z})^4 = \left[\operatorname{cis}\left(\frac{-2\pi}{3}\right)\right]^4 = \operatorname{cis}\left(4 \times \frac{-2\pi}{3}\right) = \operatorname{cis}\left(\frac{-8\pi}{3}\right)$

(De Moivre's thm)

$= \cos\left(\frac{-8\pi}{3}\right) + i\sin\left(\frac{-8\pi}{3}\right)$

$= \cos\left(\frac{-6\pi}{3} - \frac{2\pi}{3}\right) + i\sin\left(\frac{-6\pi}{3} - \frac{2\pi}{3}\right)$

$= \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

2(c)(i) $i = \text{cis}\frac{\pi}{2} = e^{i\frac{\pi}{2}}$

$i^i = (e^{i\frac{\pi}{2}})^i = e^{i^2\frac{\pi}{2}} = e^{-\frac{\pi}{2}}$

$(i^i)^2 = (e^{-\frac{\pi}{2}})^2 = e^{-\pi}$.

2(c)(ii) $e^{(i^i)} = e^{(e^{-\frac{\pi}{2}})}$

$-25e^{i^i} = 25(\text{cis}\,\pi)\, e^{(e^{-\frac{\pi}{2}})} = 25e^{i\pi}e^{e^{-\frac{\pi}{2}}}$

$= 25e^{(i\pi + e^{-\frac{\pi}{2}})}$

$\ln(-25e^{i^i}) = \ln(25e^{i\pi + e^{-\frac{\pi}{2}}})$

$= \ln 25 + \ln(e^{i\pi + e^{-\frac{\pi}{2}}})$

$= 2\ln 5 + i\pi + e^{-\frac{\pi}{2}}$

$= (2\ln 5 + e^{-\frac{\pi}{2}}) + i\pi.$

2(d) General equation: $A(x^2+y^2) + Bx + Cy + D = 0$

We will assume that $x, y \in \mathbb{R}$ (if not, redefine x, y to be so)

Then $x = \frac{z+\bar{z}}{2}$ and $y = \frac{z-\bar{z}}{2i}$.

$\left(\frac{x+iy+x-iy}{2} = x \text{ and } \frac{\cancel{x}+iy-\cancel{x}+iy}{2i} = y\right)$.

Also $z\bar{z} = (x+iy)(x-iy) = x^2 - i^2y^2 = x^2+y^2$.

Hence the general equation becomes:

$Az\bar{z} + B\left(\frac{z+\bar{z}}{2}\right) + C\left(\frac{z-\bar{z}}{2i}\right) + D = 0$

$Az\bar{z} + \left(\frac{B}{2} + \frac{C}{2i}\right)z + \left(\frac{B}{2} - \frac{C}{2i}\right)\bar{z} + D = 0$

$Az\bar{z} + \left(\frac{B}{2} - \frac{C}{2}i\right)z + \left(\frac{B}{2} + \frac{C}{2}i\right)\bar{z} + D = 0$

$\longrightarrow$

Hence, the general circle equation can be written as

$$\alpha z\bar{z} + \beta z + \bar{\beta}\bar{z} + \gamma = 0$$

with $\alpha = A$, $\beta = \frac{B}{2} - \frac{C}{2}i$, and $\gamma = D$.

This works because $B, C \in \mathbb{R} \Rightarrow \overline{\frac{B}{2} - \frac{C}{2}i} = \frac{B}{2} + \frac{C}{2}i$ so the coefficients are correct. $(\bar{\beta} = \frac{B}{2} + \frac{C}{2}i)$

3(a) $x = 2t - 3t^3 \Rightarrow \frac{dx}{dt} = 2 - 9t^2 \Rightarrow \frac{dt}{dx} = \frac{1}{2-9t^2}$

$y = te^t \Rightarrow \frac{dy}{dt} = (t+1)e^t$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{(t+1)e^t}{2-9t^2} = 0 \Rightarrow t = -1 \quad (e^t \neq 0 \ \forall t \in \mathbb{R})$$

$$(x,y)_{t=-1} = \left(2(-1) - 3(-1)^3, -1e^{-1}\right) = \left(1, -\frac{1}{e}\right).$$

For classification, use ~~first~~ second derivative test.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{(t+1)e^t}{2-9t^2}\right] = \frac{d}{dt}\left[\frac{(t+1)e^t}{2-9t^2}\right] \cdot \frac{dt}{dx}$$

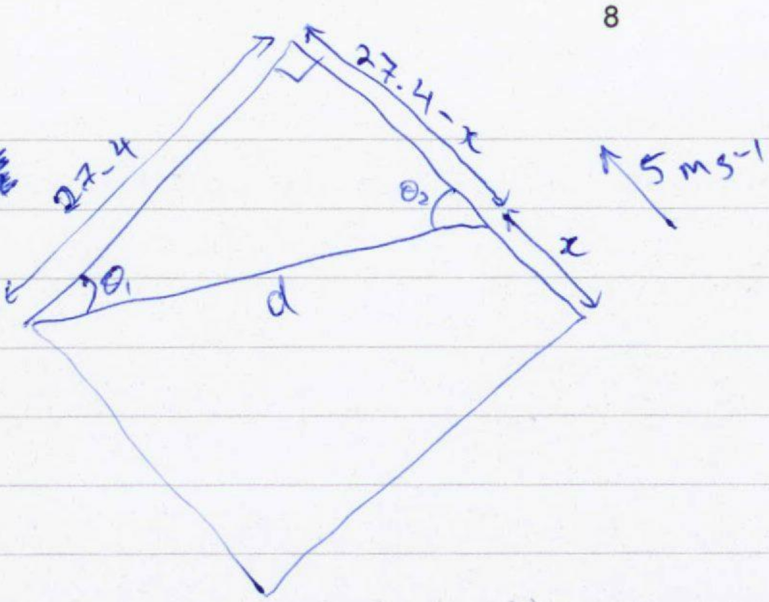
$$= \frac{(t+2)e^t(2-9t^2) - (-18t)(t+1)e^t}{(2-9t^2)^2} \cdot \frac{1}{2-9t^2}.$$

When $t=-1$, $\frac{d^2y}{dx^2} = \frac{\cancel{(-1+2)}e^{-1}(2-9) - \cancel{(18)(-1+1)e^{-1}}}{(2-9)^3} = \frac{e^{-1}}{(2-9)^2}$

$$= \frac{e^{-1}}{49} > 0, \text{ so it is a minimum.}$$

Thus the only ~~turning~~ stationary point is $\left(1, -\frac{1}{e}\right)$ and it is a minimum.

3(b)



(i) Let distance to third base be D.

By pythagoras $D^2 = 27.4^2 + (27.4 - x)^2$

Hence $\cancel{2}D\frac{dD}{dt} = -\cancel{2}(27.4 - x)\frac{dx}{dt}$

$$\frac{dD}{dt} = \left(\frac{x - 27.4}{D}\right)\frac{dx}{dt}$$

It is given that $\frac{dx}{dt} = 5$.

When $x = 10$, $D = \sqrt{27.4^2 + (27.4 - 10)^2} = \sqrt{1053.52}$

Hence $\frac{dD}{dt} = \left(\frac{10 - 27.4}{\sqrt{1053.52}}\right)(5) = -2.68039$ ms⁻¹

(ii) $\tan\theta_1 = \frac{27.4 - x}{27.4} = 1 - \frac{x}{27.4}$

$$\Rightarrow \sec^2\theta_1 \frac{d\theta_1}{dt} = \frac{-1}{27.4}\frac{dx}{dt}$$

$$\Rightarrow \frac{d\theta_1}{dt} = \frac{-1}{27.4(1 + \tan^2\theta_1)}\frac{dx}{dt}$$

$$= \frac{-1}{27.4\left(1 + \left(1 - \frac{x}{27.4}\right)^2\right)}\frac{dx}{dt}.$$

(Alternatively: at $x = 27.4$, the motion would be equivalent to circular motion with radius $= 27.4$ and v_{river} $9 = 5\ ms^{-1}$ and $\theta = \frac{v}{r}$ etc.)

When $x = 27.4$, $\frac{d\theta_1}{dt} = \frac{-1}{27.4}\frac{dx}{dt} = \frac{-1}{27.4}(5) = -\frac{25}{137}$

$= -0.18248$ rad s^{-1}.

Now since $\theta_1 + \theta_2 = \frac{\pi}{2}$,

$\theta_2 = \frac{\pi}{2} - \theta_1$

and so $\frac{d\theta_2}{dt} = -\frac{d\theta_1}{dt} = \frac{25}{137} = 0.18248$ rad s^{-1}.

3(c) Suppose the manufacturer orders steel every T days for an order quantity of n units.

Assume that the manufacturer orders exactly what he needs to make five gates per day over the time T. (for the minimal case)

Number of units per day $= \frac{n}{T} = 5 \Rightarrow n = 5T$.

~~Average daily cost of delivery & storage = order cost ... T~~

As each day passes from the time of order/delivery (assume delivery is instant as no delivery speed is given), the number of units decreases by 5.

1 cycle.

Day 1	Day 2	Day 3	...	Day T
5T units	5T−5 units	5T−2x5 units		5 units

Hence the total cost of storage over 1 cycle is

$10(5 + 10 + 15 + \dots + 5T)$

$= 10(5)(1+2+3+\dots+T) = 50(1+2+3+\dots+T)$

$= 50\,\frac{T(T+1)}{2} = 25T(T+1)$.

Quick proof of $1+2+3+\ldots+N = \frac{N(N+1)}{2}$:

$1+2+3+\ldots+N = \binom{N+1}{2}$ ("number of handshakes" combinatorial concept)

but $\binom{N+1}{2} = \frac{(N+1)!}{2!(N+1-2)!} = \frac{\cancel{(N-1)!}\,N(N+1)}{2\cancel{(N-1)!}} = \frac{N(N+1)}{2}$. #

Total cost of storage over 1 cycle $= 25T(T+1)$

Average daily cost of storage over 1 cycle $= \frac{25\cancel{T}(T+1)}{\cancel{T}} = 25(T+1)$

Hence, average daily cost of delivery and storage:

$$C = \frac{5000}{T} + 25(T+1).$$

$$\frac{dC}{dT} = -\frac{5000}{T^2} + 25 = 0$$

$$\Rightarrow \frac{5000}{T^2} = 25$$

$$\Rightarrow T^2 = \frac{5000}{25} = 200$$

$$\Rightarrow T = \sqrt{200} = 14.14\ldots \quad (T>0)$$

(Check minimum: $\frac{d^2C}{dT^2} = \frac{10000}{T^3} > 0$ when $T>0$ ✓)

But T needs to be a whole number so we check either side of 14.14:

If $T=14$, $C = \frac{5000}{14} + 25(14+1) = 732.14 \ \frac{\$}{\text{day}}$

If $T=15$, $C = \frac{5000}{15} + 25(15+1) = 733.33 \ \frac{\$}{\text{day}}$

So, we see that $T = 14$ days yields the minimum, and a total of $n = 5T = 70$ units should be ordered each time.

~~Area = p+q = (q−p) = 2(−p)~~

4(a) Find the intersection points: $x^2 - 2x = 6x - x^2$

$\Rightarrow 2x^2 - 8x = 0 \Rightarrow x^2 - 4x = 0 \Rightarrow x \in \{0, 4\}$

~~Area = $\int (6x-x^2) - (x^2-2x)$ Notice the region of x^2-2x which dips below the x-axis, this happens over $0 \le x \le 2$.~~

~~Hence, to find the absolute area of the grey region~~

$$\cancel{A = \int_0^4 (6x-x^2)-(x^2-2x)\,dx - 2\int_0^2 x^2-2x\,dx}$$

$$\cancel{= \int_0^4 8x-2x^2\,dx + \int_0^2 4x-2x^2}$$

$$\cancel{= \left[4x^2-\frac{2x^3}{3}\right]_0^4 + \left[2x^2-\frac{2x^3}{3}\right]_0^2}$$

$$\cancel{= \frac{64}{3} + \frac{8}{3} = \frac{72}{3} = 24 \text{ units}^2}$$

$$\text{Area} = \int_0^4 (6x-x^2)-(x^2-2x)\,dx$$

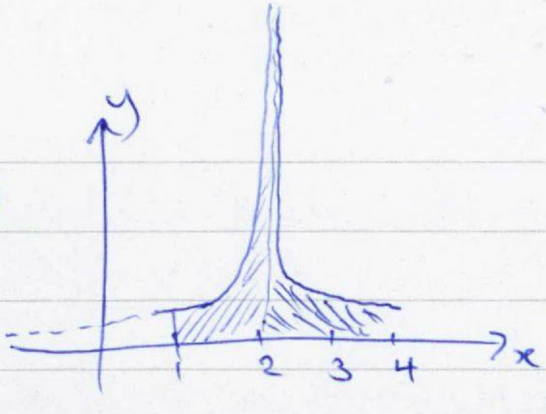
$$= \int_0^4 8x - 2x^2\,dx$$

$$= \left[\frac{8x^2}{2} - \frac{2x^3}{3}\right]_0^4$$

$$= \frac{64}{3} \text{ units}^2$$

top scholar again baby

4(b)



Due to the discontinuity over $x=2$,
we must evaluate the (improper) definite integral $\overbrace{\int_1^4 \frac{dx}{(x-2)^{2/3}}}^{I}$ as

$$I = \underbrace{\int_1^2 \frac{dx}{(x-2)^{2/3}}}_{I_1} + \underbrace{\int_2^4 \frac{dx}{(x-2)^{2/3}}}_{I_2}$$

$$I_1 = \lim_{k \to 2^-} \int_1^k (x-2)^{-\frac{2}{3}} \, dx$$

$$= \lim_{k \to 2^-} \left[(x-2)^{\frac{1}{3}} \cdot 3 \right]_1^k$$

$$= \lim_{k \to 2^-} \left[3(k-2)^{\frac{1}{3}} - 3(-1)^{\frac{1}{3}} \right]$$

$$= \left[\cancel{3(0)} - 3(-1) \right] = 3.$$

$$I_2 = \lim_{k \to 2^+} \int_k^4 \frac{dx}{(x-2)^{2/3}}$$

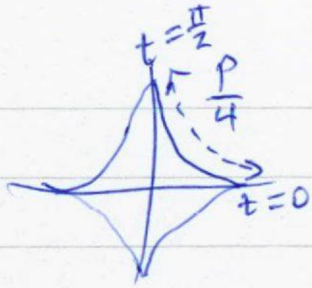
$$= \lim_{k \to 2^+} \left[3(x-2)^{\frac{1}{3}} \right]_k^4$$

$$= \lim_{k \to 2^+} \left[3(4-2)^{\frac{1}{3}} - 3(k-2)^{\frac{1}{3}} \right]$$

$$= \left[3\sqrt[3]{2} - \cancel{3(0)} \right] = 3\sqrt[3]{2}$$

Hence $I = I_1 + I_2 = 3 + 3\sqrt[3]{2}$.

4(c)



$x = \cos^3 t \Rightarrow \frac{dx}{dt} = 3\cos^2 t\,(-\sin t)$

$y = \sin^3 t \Rightarrow \frac{dy}{dt} = 3\sin^2 t\,(\cos t)$

Hence $\frac{P}{4} = \int_0^{\frac{\pi}{2}} \sqrt{(3\cos^2 t\,(-\sin t))^2 + (3\sin^2 t \cos t)^2}\, dt$

$\Rightarrow P = 4\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\; dt$

$= 4\int_0^{\frac{\pi}{2}} \sqrt{9\cos^2 t \sin^2 t\, \cancel{(\cos^2 t + \sin^2 t)}}\; dt$

$= 4\int_0^{\frac{\pi}{2}} |3\cos t \sin t|\, dt$

$= 4\int_0^{\frac{\pi}{2}} 3\cos t \sin t\, dt$ (as $\cos t, \sin t > 0$ over $0 \le t \le \frac{\pi}{2}$)

$= \int_0^{\frac{\pi}{2}} 12\cos t \sin t\, dt$

$= \int_0^{\frac{\pi}{2}} 6\sin 2t\, dt$

$= \left[-3\cos 2t\right]_0^{\frac{\pi}{2}}$

$= \left[-3\cos \frac{\cancel{2}\pi}{\cancel{2}}\right] - \left[-3\cos 0\right]$

$= -3(-1) + 3$

$= 3 + 3$

$= 6$ units.

4(d) $[f(x)]^2 = \int_0^x \left(f(t)^2 + f'(t)^2\right) dt + 2023$

Differentiate both sides w.r.t. x:

$2[f(x)]f'(x) = f(x)^2 + f'(x)^2 + 0$

$f(x)^2 - 2f(x)f'(x) + f'(x)^2 = 0$

$\left(f(x) - f'(x)\right)^2 = 0$

$f(x) = f'(x)$

$\int 1\, dx = \int \frac{f'(x)}{f(x)}\, dx$

$x + c = \ln|f(x)|$

$|f(x)| = e^{x+c} = Ae^x$

Note: the potential "$\ln|x| + c$" error does not arise due to $x \geq 0$ given as the domain in the question.

$f(x) = Ae^x$.

Then $\left(\int_0^x A^2e^{2t} + A^2e^{2t}\, dt\right) + 2023 = A^2e^{2x}$

$\int_0^x 2A^2e^{2t}\, dt + 2023 = A^2e^{2x}$

$\left[A^2e^{2t}\right]_0^x + 2023 = A^2e^{2x}$

$\cancel{A^2e^{2x}} - A^2\cancel{e^0} + 2023 = \cancel{A^2e^{2x}}$

$A^2 = 2023$

$A = \pm\sqrt{2023} = \pm\sqrt{17^2 \times 7} = \pm 17\sqrt{7}$.

However, if $f(x) = \sqrt{\cdots}$,

then $f(x) \geq 0$, so we must reject $A = -17\sqrt{7}$.

This leaves $\boxed{f(x) = 17\sqrt{7}\, e^{x}}$.